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LETTER TO THE EDITOR

The transverse structure of a spin chain with Dzyaloshinskii–Moriya-type interaction

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Abstract. The transverse helical spin structure of the uniaxial spin ($s = \frac{1}{2}$) chain with exchange–relativistic non-uniform interaction is found. The magnetic behaviour of this system is considered.

There is now growing interest in low-dimensional magnetic systems. The magnetization and the magnetic susceptibility of such spin systems are determined mostly by relativistic interactions. Among the low-dimensional systems there are systems that have exchange–relativistic coupling of Dzyaloshinskii–Moriya type. Theoretical studies of these systems usually describe the spin oscillations about their classical equilibrium positions. One can see from these approximations that the weak ferromagnetism phenomenon (i.e. non-zero spontaneous magnetization) is found. Recently the author of this letter [1] and Alkaraz and Wreszinski [2] found the ground state energy of the spin chain with Dzyaloshinskii–Moriya coupling. Eckle and Hamer [3] then calculated the finite-size ground energy of the anisotropic Heisenberg chain in an external magnetic field for twisted boundary conditions.

In this paper I investigate theoretically the ultra-quantum limit ($s = \frac{1}{2}$), for which exact solutions can be studied, and show that the Dzyaloshinskii–Moriya coupling leads to helical transverse spin structure for such one-dimensional uniaxial spin systems. As is known, the one-dimensional spin ($s = \frac{1}{2}$) system with Dzyaloshinskii–Moriya interaction does exist (see, for example, [4]). The Hamiltonian of the system considered has the form

$$\mathcal{H} = -\tfrac{1}{2}\sum_n [J(\sigma_n^x\sigma_{n+1}^x + \sigma_n^y\sigma_{n+1}^y) + D(\sigma_n^x\sigma_{n+1}^y - \sigma_n^y\sigma_{n+1}^x) + J_z\sigma_n^z\sigma_{n+1}^z] \qquad (1)$$

where J and J_z are the exchange constants, D is the Dzyaloshinskii–Moriya interaction constant, and $\sigma_n^{x,y,z}$ are the Pauli operators. With the help of the standard Bethe

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procedure [5] we derive the set of equations for the quantum numbers p_j and the energy of the state with the $(N-m)$th value of the z-projection of the total spin:

$$\exp(\mathrm{i}p_j N) = (-1)^{m-1} \exp\left(-\mathrm{i} \sum_{k=1}^{m} \theta(p_j, p_k)\right) \tag{2}$$

$$E = -(NJ_z/2) + (J^2 + D^2)^{1/2} \sum_{j=1}^{m} (B - \cos(p_j + \xi)) \tag{3}$$

where N is the number of sites. Expression (2) imposes cyclic boundary conditions. Here (in (2), (3))

$$\theta(p_j, p_k) = 2\tan^{-1}\{[B\sin((p_j - p_k)/2)]/[\cos(\xi + (p_j + p_k)/2) - B\cos((p_j - p_k)/2)]\} \tag{4}$$

$$B = J_z/(J^2 + D^2)^{1/2} \qquad \tan\xi = -D/J. \tag{5}$$

One can see that the Dzyaloshinskii–Moriya coupling, as usual, affects the shifts of the quasimomenta p_j (the magnitudes of these shifts are equal to ξ).

Let us find and treat the ground state energy of the system considered.

(i) If $B \geqslant 1$, the ground state of the spin chain is ferromagnetic, and its energy is equal to $\varepsilon = E/N = -J_z/2$.

(ii) If $B < 1$, the ground state of the uniaxial spin chain corresponds to the zero value of the z-projection of the total spin, i.e. $m = N/2$ [6]. Taking the limit $N \to \infty$ we have for the ground state, if $-1 < B < 1$,

$$\varepsilon = J_z + (J^2 + D^2)^{1/2} \sin\lambda \times \int_{-\infty}^{\infty} \mathrm{d}x \sinh((\pi - \lambda)x)/\sinh(\pi x)\sinh(\lambda x) \qquad \cos\lambda = -B. \tag{6}$$

If $B = 1$, the ground state energy has a form

$$\varepsilon = (J^2 + D^2)^{1/2}(\ln 4 - \tfrac{1}{2}). \tag{7}$$

Finally, if $B < -1$, the ground state energy is equal to

$$\varepsilon = J_z + (J^2 + D^2)^{1/2} \sinh \sum_{n=1}^{\infty} \exp(-2n\nu)\tanh(n\nu) \qquad \cosh\nu = -B. \tag{8}$$

The average values are equal to zero ($\langle\sigma_n^{x,y}\rangle = 0$) in the ground state. We know (see [7]) that the plane spin structure of the chain is determined by correlators such as $\langle\sigma_n^{x,y}\sigma_{n+r}^{x,y}\rangle$. Let the projection of the spin in the nth site on the axis which makes the angle φ_n with the axis x be equal to unity; the projection of the spin in the $(n+r)$th site on the axis which makes the angle φ_{n+r} with the axis x is equal to unity too. The maximum probability then occurs at the angles φ_n and φ_{n+r} given by:

$$\tan(\varphi_n - \varphi_{n+r}) = \langle\sigma_n^x\sigma_{n+r}^y - \sigma_n^y\sigma_{n+r}^x\rangle / \langle\sigma_n^x\sigma_{n+r}^x + \sigma_n^y\sigma_{n+r}^y\rangle. \tag{9}$$

To treat the nearest-neighbour ordering, it is enough to consider the case $r = 1$ only. With the help of (6)–(8) we obtain

$$\tan(\varphi_n - \varphi_{n+1}) = \begin{cases} 0 & \text{if } B \geqslant 1 \\ D/J & \text{if } B > 1. \end{cases} \tag{10}$$

Equation (10) means that in the non-trivial case where $J_z < (J^2 + D^2)^{1/2}$, the helical transverse structure of the spin chain is obtained. The shift of the helical structure is determined by the constant of Dzyaloshinskii–Moriya coupling, as is given by classical consideration of the spin system [8].

The ground state energy is ferromagnetic for $B \geqslant 1$ at any value of the magnetic field h directed along the z axis and for $B < 1$ for $h > [(J^2 + D^2)^{1/2} - J_z]/\mu = h_c$ (μ is the Bohr magneton). The z-projection of the total spin (yN) has its nominal value. Naturally, there is no transverse spin structure in these cases.

If $B \geqslant 1$ the ground state energy is equal to $\varepsilon^h = \varepsilon - \mu h$. For $B < 1$, if $1 - y \ll 1$ (the value of the magnetic field is little less than h_c) the ground state energy is equal to

$$\varepsilon^h = -(J_z/2) + \mu h y - [(J^2 + D^2)^{1/2} - J_z](1-y) + \pi^2(1-y)^3/24 + \ldots. \tag{11}$$

If $y \ll 1$ ($h \to 0$) for $B < -1$ we have

$$\varepsilon^h = \varepsilon - \mu h y + (J^2 + D^2)^{1/2} \sinh[c_0 y + (\pi^2 c_2 y^3/3c_0) + \ldots]. \tag{12}$$

The coefficients c_n are defined from

$$c_0 + c_2\alpha^2 + c_4\alpha^4 \ldots = \sum_{n=-\infty}^{\infty} (-1)^n \cos(n\alpha)/2\cosh(n\nu).$$

For $B = -1$ the ground state energy is determined by

$$\varepsilon^h = \varepsilon - \mu h y - (J^2 + D^2)^{1/2}\pi^2 y^2/4 + \ldots. \tag{13}$$

Finally, for $-1 < B < 1$ we have

$$\varepsilon^h = \varepsilon - \mu h y - (J^2 + D^2)^{1/2} 2\pi(\pi - \lambda) y^2 \sin\lambda/8\lambda + \ldots. \tag{14}$$

It can be concluded from (11)–(14) that for $h < h_c$ if $B < 1$ the transverse spin helical structure is obtained. The helical structure's shift is the same as in the absence of a magnetic field.

Using the method of [9–11] we derived the sets of equations describing the thermodynamics of the spin chain with Dzyaloshinskii–Moriya coupling. With the help of those equations one can see that (10) still holds for the case $T \neq 0$ (T is the temperature) if $B < 1$ at any value of the magnetic field.

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